Mathematical literacy for everyone using arithmetic games

L Pareto

Media & Design, University West, Trollhättan, SWEDEN

lena.pareto@hv.se

www.hv.se

ABSTRACT

An innovative mathematics game shown to be effective for low-achieving mainstream students is tested in special education for learners with intellectual disabilities. The game relies on a graphical, intuitive representation for numbers and arithmetic operations to foster conceptual understanding and numbers sense, and provides a set of 2-player games to develop strategic thinking and reasoning skills. The game runs on computers and interactive white boards, and as an augmented reality application at a science centre. We compare its use in special education and mainstream education with respect to usage, performance levels and learning gain. The game has been used by teachers in special educations, with gains in mathematical understanding, strategic thinking and communication skills as effects.

1. INTRODUCTION

Mathematical literacy is crucial for handling everyday life situations, such as being on time, paying bills, using maps, reading time tables, and comprehending expiry dates (Gregoire & Desoete, 2009). For students with severe learning disabilities the world of numbers may never become accessible, as repeatedly trying to solve problems without an understanding of the underlying concepts creates passive learners and often also results in learned helplessness (Miller & Mercer, 1997). The impact of mathematical skills on employment prospects is even bigger than the influence of poor reading skills (Dowker, 2005). Years of failure in mathematics, can seriously handicap daily adult life (Garnett, 1998). Yet, research on mathematical disabilities is still comparatively scarce (Gregoire & Desoete, 2009).

Research suggests that a mathematical curriculum for students with learning disabilities should focus on "big ideas": a few important concepts (e.g., basic arithmetic) should be taught to mastery rather than numerous skills superficially (Gersten & Chard, 1999). Still, these authors claim, special education instruction continues to focus on computation, in particular exact arithmetic calculations (Rousselle & Noël, 2008) rather than mathematical understanding and estimation. For special education, the following practices are recommended: encourage and practice to "talk math" (Garnett, 1998); use motivational practices such as games (Garnett, 1998); use board-games and other (physical) manipulatives and instructional software (Lock, 1997). Recommendations that, a decade later, were still valid (Evans, 2007).

Our instructional software, a card- and board game, targets conceptual understanding in arithmetic, such as the meaning of addition, and higher-order cognitive skills, such as reasoning and strategic thinking. Students are encouraged to discuss the game (i.e., talk math) while trying to choose good cards. The game is designed to serve as an alternative for students with mathematical difficulties or intellectual disabilities (Pareto, 2005) but as a challenging and useful complement in mainstream education. In other words, it follows a universal design approach rather than an assistive technology approach (Shneiderman, 2000). The game has proved effective for improving students' conceptual understanding in basic arithmetic, and for promoting better self-efficacy beliefs in mainstream education, with the strongest effect for low-achieving students (Pareto et al, 2011). The game has also been used in special math classes within the standard Swedish curriculum, where students receive additional support by special teachers (Nilsson & Pareto, 2010).

In this study, we investigate whether the game can be used effectively also in special education. We are interested in whether its usage in this context differs from that of mainstream classroom use, and whether game-play behaviour, game performance and learner progress differ between the two user groups. We also investigate whether an augmented reality version of the game, enhances game experience and learning.

2. SPECIAL EDUCATION LEARNERS AND MATHEMATICS

Special education for students unable to follow mainstream curricula due to developmental disabilities or permanent brain injuries has existed since 1968 in Sweden. Today, the special education school is a 9-year mandatory school (just as the standard school) with a special, reduced curriculum. Reasons for attending special education include various intellectual disabilities and also severe ADHD and autism. The Special Education includes curricula for students with mild disabilities (normally defined as an IQ below 70), and *training school* for students with moderate or severe disabilities (IQ below 50). However, the IQ level is not the only factor determining if a student is entitled to special education: prior to enrolment, a thorough individual investigation of the student's psychological, social, pedagogical and medical ability to follow the standard curricula is conducted, and consent is sought by the parents.

A national organization, The Swedish Agency of Special Needs Education, has the aim to ensure that children, young people and adults with disabilities will be able to develop and receive an education based on equality, participation, accessibility and companionship. Such view originates from Vygotsky (1978), who argued that the main objective in the field of special education should be the creation of what he called a ”positive differential approach”, that is, the identification of a disabled child from the point of strength rather than a disability. He suggested for example to measure the level of overall independence in children with mental retardation, rather than the degree of feeblemindedness, which is adopted today. Gindis (1995) argues that Vygotsky's view is the most comprehensive, inclusive and humane practice of special education in the 21st century. In Sweden, the ambition is to have an inclusive school and integrate special education students in standard classes as much as possible, but this is obviously not a reasonable solution for all students.

Intellectual disability is a broad concept encompassing various intellectual deficits, including mental retardation (MR), various specific conditions such as specific learning disability, and problems acquired later in life through acquired brain injuries. Since we study mathematics education, we will go into specific difficulties related to learning disabilities in mathematics. Munro (2003) has identified the following difficulties: difficulty in using mathematical concepts in oral language, difficulty manipulating concrete material such as enumerating objects, difficulty reading and writing mathematical symbols, difficulty understanding mathematical ideas and relationships, and difficulty performing mathematical operations. The different types of difficulties can occur in isolation or in combination (Munro, 2003).

Arithmetic is the base of mathematics. Counting is the most basic skill in arithmetic, since it relates real world quantities to numbers. There are basic rules that need to be understood to be able to count effectively: things should be counted once; the number of the last counted object is the magnitude; the counting order is irrelevant, which is typically misunderstood (Geary, 1999). Even infants have a direct, intuitive perception of the magnitude of small collections of objects (Butterworth, 2005). Most adult can grasp collections up to 7-8 objects in this direct, intuitive manner. This mental process, referred to as subitization, is quite different from counting. A disability in subitizating means not seeing the “threeness” of three objects, thus even small collections of objects must be counted. Children must also learn to associate the three representations of a number: the symbol “2”, the word “two” and the quantity (magnitude) “2”. Mapping number words onto the representations of these quantities is a difficult task, which can be especially challenging for children with math disabilities who also experience reading difficulties (Berch, 2005). Many students with disabilities can learn the quantitative aspects of mathematic concepts better than the symbolic counterpart. Studies suggest that the two representations are functionally independent (Munro, 2003), thus should not be addressed together. Our game design takes this into account: if the root of the problem is lack of number-sense, this has to be addressed before any further progress can be made (Ranpura, 2000). Finally, students with learning disabilities have difficulties abstracting principles from experiences (Geary, 1993), so opportunity for extensive and prolonged experience with mathematics is needed.

3. THE EDUCATIONAL ARITHMETIC GAME

It is far from evident how to design a game environment that fosters deep mathematical understanding (Moreno & Meyer, 2005). Our approach is to provide 1) a graphical model simulating arithmetic behaviour; 2) a set of two-player games supporting collaboration and competition; and 3) intelligent, teachable agents which can be taught to play the games.

Our educational game teaches arithmetic, specifically base-ten concepts, including place-value, carrying and borrowing, and estimating sets. Notice that base-ten concepts is a major stumbling block for many elementary school children (Carpenter, Fennema & Romberg, 1993), not only children with learning difficulties. The four arithmetic operations are covered, using negative as well as positive integers. All mathematical concepts are represented graphically: integers are coloured blocks; arithmetic operations are animated actions involving those blocks. All mathematical rules are built into, and thus ensured, by the model. This allows properties and relations to be discovered and explored by the students, since all possible actions are guaranteed to be mathematically valid. The goal is to learn by proxy of doing and by reasoning and making choices and estimations, which is in stark contrast to performing computations and studying concepts in isolation (Case & Okamoto, 1996).

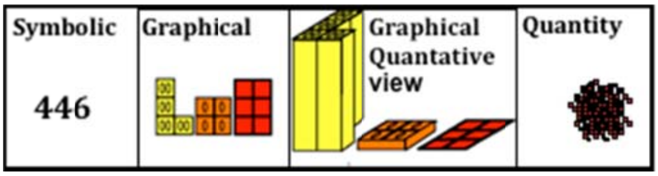

Figure 1. *Graphical representation of integers.*

The graphical representation of integers is coloured blocks and boxes of different heights, as a metaphor for numbers (see Figure 1). Ones are represented as flat red blocks; tens as orange boxes marked with a single zero; hundreds as yellow, taller boxes with two zeros. An orange box contains ten red flat blocks stacked together; a yellow box contains ten stacked orange boxes.

The set of two-player games are based on the graphical model. In these games, players act arithmetic operations using a set of cards. Players take turns, choosing one card, until all cards are played. Although the game, in effect, constitutes a sequence of computations, the task facing the players is not to do the computations but to choose good cards to play according to the game's particular goal, such as maximizing the overall number of carries or the number of zeroes in each turn. Game goals are designed to reveal important properties of arithmetic. For instance, in Figure 2 (top left), the goal is to maximize carries. Here, a card with 6 red blocks is placed above the blue game board already containing 8 red blocks, which mathematically means 8 + 6. This situation occurred since the current player plays addition and just chose the card 6. Now, the computation 8+6 is carried out by the system, following a low-stress algorithm of explicit packing where the blocks from the card are added one by one to the game board until the encircled area is filled with 9 blocks, then the 10^{th} block lands in a "packing area" (the orange arrow illustrating a 10-box above the board) and the 9 blocks on the board are also moved one by one filling the box with ten blocks. When the packing is complete, the orange box falls down in the turquoise 10^{th}-compartment to the right, and the remaining 4 red blocks will now fit, and the computation is completed with the following result: 1 orange ten and 4 red ones, which denotes 14. Such low-stress algorithms can be crucial for learners experiencing difficulties, since they are more explicit, reduce anxiety, and help to conceptual understanding (Lock, 1996).

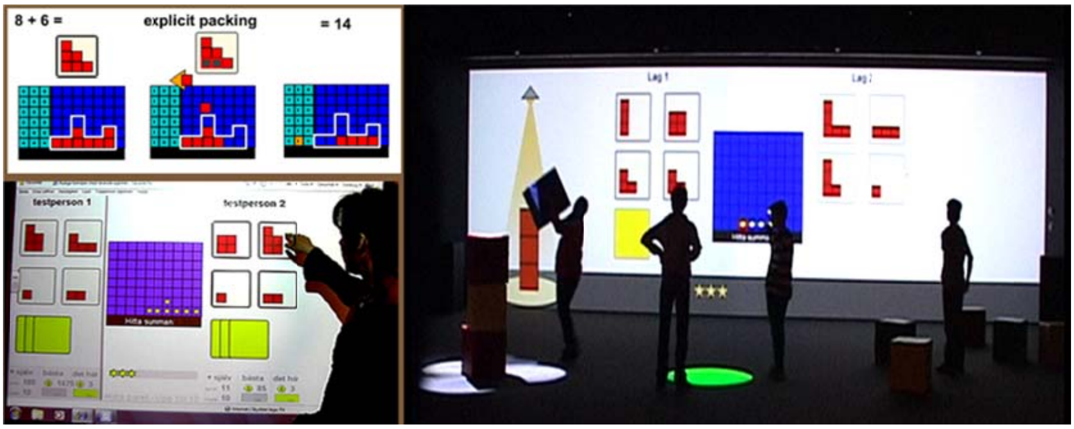

Figure 2. *Explicit carrying operation as packing blocks into boxes (top left), game on interactive white board (bottom left) and game as augmented reality application in science center (right).*

In Figure 2 (bottom left), another game is shown. Here, the goal is to find a pair of cards (one from each player) matching a given goal, in this case the sum 7 (seven stars on the game board that should be filled with red blocks). This game is collaborative and encourages players to reason and discuss alternatives. One solution is to take the card 5 from the right side and 2 from the left, another to take 2 from the right and 5 from the left. The other cards on the right (4 and 1) do not have a matching card fulfilling the goal.

The game currently runs on three platforms: ordinary computers, interactive white boards (Figure 2, bottom left) and an interactive augmented reality 3D-cinema at a regional science centre (Figure 2, right). In the 3D-cinema, users interact with physical objects connected to a virtual world shown on a 4 meter tall and 12 meter wide scene in 270° wide angle. Here, the study participants 1-4 collaborate by moving physical cubes into a lit-up area to select a corresponding card on the screen. A virtual representation of the 3 cubes is shown on the left of the screen, partly covered by a user carrying the 4th cube towards the light. The cubes contain microchips detected by a sensor and mirrored on the screen; a sensor in the green-lit area detects movements. To select a card with 4 blocks, 4 cubes should be moved to the building area and then someone (or something) must cross the green area.

The game addresses math disability problems in several ways. It provides a constructive, analogical representation, which support understanding the counting principle. The graphical representation of various block-formations encourages different kinds of counting strategies, so that the order-irrelevance principle is explicitly practiced in the model. It also provides opportunity to practice subitization since every digit is represented by a group of blocks proportional to the magnitude. If the magnitude cannot be "seen", the blocks can be counted. For further details see (Pareto, 2005). In the augmented reality version of the game, the sensation of magnitude is further enhanced by having to operationally moving cubes one at a time to experience the sensation that moving 8 cubes are much more cumbersome than moving 2.

Further, the game gives substantial training of causal reasoning, a basic cognitive process that underpin all higher-order activities. (Research on instructional methods for supporting causal reasoning is scarce (Jonassen & Ionas, 2008)). In order to play well, the effect of each card must first be considered, then the alternatives should be valued against each other. The first part requires mental computations or estimations; the second part requires strategic thinking and the ability to reason and judge. Previous research supports our approach: Students with mental retardation can learn to employ cognitive strategies effectively with strategy instruction (Butler et al., 2001). Such strategy instruction promoted not only mathematics performance, but also student independence. Also, other technological innovations (e.g., digitized text combined with scaffolds to assist comprehension) have enabled teachers to provide students with disabilities access to complex concepts and to engage them in higher order thinking (Brownell et al., 2010).

4. THE STUDY

Currently, 7 schools, 60 elementary teachers, and over 900 students in grade 1-8 in a municipality in West Sweden are enrolled in programs financed by the Swedish National School Board or Swedish National Agency for Special Needs. The educations use the game as an educational tool in their regular mathematics lessons, with the objective to improve national standard math comprehension test results. Eight of these students, all enrolled in a Special Education training school, participated in the study. (See Table 1),

Table 1. *Participants in study.*

Special Education students	Class level	Disability	Month played	Strategic game data	Match pair game data	Class room observation	Augmented reality observation	Teacher interview
1	7-8	Intellectual & language	15	X	X		X	X
2	7-8	Intellectual & socially	15	X	X		X	X
3	7-8	Intellectual & language	15	X	X		X	X
4	7-8	Intellectual & language	15	X	X		X	X
5	9	Asperger syndrome	12	X	X		X	X
6	5-6	Down syndrome	4	No data	X	X		
7	5-6	Down syndrome	4	No data	X	X		
8	5-6	Down syndrome	4	No data	X	X		

All the participants except student 5 qualify for training school curriculum, i.e., are judged to have a moderate or severe intellectual disability. Down syndrome is associated with a delay in cognitive ability and physical growth, and a large proportion of individuals with Down syndrome have a severe degree of intellectual disability (Grant at al., 2010). Student 5 is diagnosed with Asperger syndrome, an autism spectrum disorder that is characterized by significant difficulties in social interaction and restricted interests. It differs from other autism spectrum disorders by its relative preservation of linguistic and cognitive development. Atypical use of language is common (McPartland & Klin, 2006).

We engaged the special teachers and their students of training schools in our municipality, who have used the game for the past 2 years. Students 1 to 4 have used the game for the longest time and are still active users. Student 5 used an earlier version of the game two years ago; for him, the main purpose was to practice social and collaborative skills rather than mathematics. Students 6 to 8 have used the game since this spring.

The study included several data sources. *Game playing logs* for all participating students and for about 300 mainstream students in grade 3 to 8 to compare with; in these logs, we can trace playing behaviour and calculate various measures of playing performance and progression. *Qualitative data from observations and interviews*; we have observed the first two game playing sessions for students 6-8, which took place in a classroom setting; these observations focused on teachers' and students' behaviour and interactions during play, and were documented in detail. *Notes from a reflection session* with a teacher. *Recordings* from a first test of the augmented reality version of the game in a science centre, where students 1-4 participated as players and student 5 as tutor: a video recorded from from the back to capture the screen and overall interactions and from the front-corner to catch facial expressions and gestures; a separate recording of the players' speech, using individual microphones in order to capture all communication. *Interviews* with two teachers of students 1-5 concerning 1) the students' ability profiles in general and their mathematical level before starting with the game, 2) the teachers' assessment of individual progressions, and 3) their perception of learning benefits advocated to the game; this 2-hour interview was recorded and transcribed. Unfortunately, the semester ended before we had time to interview the teachers of students 6-8 and to bring these students to the science centre.

5. RESULTS

5.1 Student ability profiles before play according to teachers' assessments (from teacher interviews)

Student 1 has an unusual disease affecting his language (talk is slurred) and his fine motor skills (which are poor). At the beginning of the study, he was enrolled in the 7th grade, but should have been 8th grader according to age. His mathematical ability was higher than that of his classmates (student 2-4): he could perform simple computations (which his classmates could not). He did however not understand what he was doing, and did not even pass national tests for grade 3. *Student 2* has a mental age of about 3, and his knowledge level is accordingly very low for grade 7, as he has problems with number sense up to 10. He can communicate well, but seeks and needs the support of adults for many activities. *Student 3* has severe stuttering problems, so communication is hard. He could read simple texts, perform simple addition and subtraction but he had no numbers sense so procedures were performed mechanically with lack of understanding. *Student 4* is at a very low intellectual level. He is bi-lingual but cannot read or write, and a year ago he used only a few words in either language. He managed to point and count physical objects, but could not perform computations like 1+3. He had no number sense and his self-efficacy was extremely low. *Student 5* participates for social skills training by using this collaborative mathematical game. When he started this program he refused to communicate with anyone or participate in any learning activity; he had no tolerance of or empathy for peers less intellectually competent than himself. In the beginning of playing, he voluntarily initiated a discussion with a peer for the first time ever in a classroom situation.

5.2 Performance data from the Match-Pair games

All participants except student 5 played the rather newly developed Match-Pair games.

In figure 3, we can see the performance levels of how many of the matching pairs that players in a particular grade have found in average in the mainstream condition, compared to the performance levels of each of our participants. All included students have played at least 20 rounds. The performance measure is percentage found pairs as visualized by the bars. The four measured games are shown in groups on the y-axis. All games have the same goal in each round to find two matching cards, one from each player, that together make a given sum or difference. The games only differ in number range: from the numbers range 1-10, to the numbers range 1-20 where carrying and borrowing are involved, to the range 1-100, and finally to the range -10 to 10 where both positive and negative integers are involved in the sums and differences.

The first 5 bars denote mainstream students in 1st to 5th grade; the middle 4 red, striped bars denote study participant in grade 7-8, and the last yellow, dotted bars denote participants in grade 5-6 (who only played leftmost, simplest game). We can see that student 1 is performing better in more challenging games, and performs at the same level or above 5th graders. Student 2 and 4 only played the match-pair games up to 20, but in these games, they perform better than the average 5th grader. Student 3 performs better than the average 5th grader on the games involving positive integers in range up to 100, but not on the game involving negative integers.

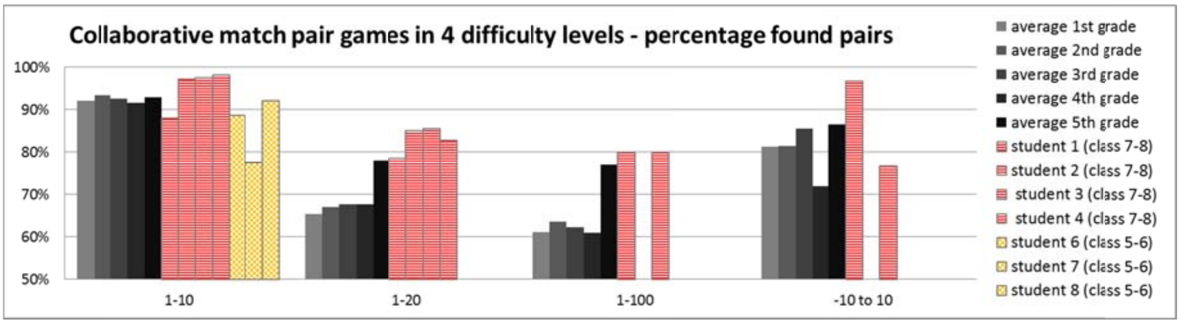

Figure 3. *Performance data from Match-Pair game comparing mainstream and special education students*

5.3 Behaviour data from 2 classroom observations

The two first lessons with the easiest game Match-Pair 1-10 for student 6 to 8 have also been observed. The first lesson proceeded as follows: the three students were placed on chairs in front of the interactive white board, where the teacher and an assistant showed the game several rounds. They emphasized the game idea by making strong gestures accompanying the flashing star (scoring feedback), and showed great enthusiasm and joy. After some rounds, they asked the students to suggest cards to play in order to involve them further. Student 6 and 7 grasped the idea and did suggest matching cards quite often. However, when these two students played themselves shortly after, student 7 where not anymore as confident in choosing, and after some mistakes she refused to play more. Student 6 and 8 played instead, and after some time student 8 also managed to choose a matching card from time to time, and they ended up finding 7 of 10 pairs in their best round. However the idea of collaborating was perhaps the most difficult aspect of the game, and student 6 continued exclaiming "*I won*" every time they received a star despite being corrected to "*the two of you got a star*" by the teacher. In a short reflection session afterwards, the teacher commented that collaborating is difficult for these students, so she was pleased they could practice that in the game. She also explained that student 7 usually wanted to succeed so badly that she refused to do activities where this was jeopardized.

In session 2, the setup was the same. Student 6 and 8 first played together, then each student played the teacher or assistant. Student 7 still refused to play herself, but did participate somewhat from her chair. The players have grasped the mathematical part well, and received high scores most of the time. They were engaged and happy while playing and to the teacher's surprise the students were able to concentrate 10 minutes beyond the scheduled time. Student 8 concluded the lesson with "*This was a good game*".

5.4 Behaviour data from the augmented reality prototype test observation in the science centre

The study participants 1-5 were the first to test the new version of the game in the science centre's augmented reality cinema. The ported game is the same Match-Pair 1-10 as on the other platforms, but where cards are chosen by moving cubes (recall Figure 2, right picture). If there is a card with the same number of blocks as cubes in the building area, then the card is played and the turn goes to the other player. If there are no cards matching the cubes, nothing happened for now (error feedback will be provided but that part was not finished for this first test) and this gave rise to some confusion, but also to some moments for reflection. Students 1-4 tested the game as players, and student 5 as tutor helping the others to play the game.

First student 2 and 4 played. The tutor guided them a little bit, but rather soon they understood the idea. Student 4 takes many initiatives, and is clearly collaborating with his partner, asking "*Where should I put them*" and "*Which should I take, the 3*"? Student 2 takes the lead and is receiving some help from the tutor student, e.g. when the goal is 6 student 2 says "*I take 3 and he takes 3*" (which is properly calculated, but there is no card with 3 blocks among his alternatives). The tutor responds "*You don't have a card with 3, you must take the card with 4*". The game proceeds and soon the two players make their choices without guidance. Student 4 evidently enjoys playing; he walks around smiling and singing while moving the cubes. Then student 1 and 3 continues, and since they already watched a game they know pretty much how to do. Student 3 moved most of the cubes, while student 1 mainly handled the green light selections.

Finally all four students were playing together in pairs, without the tutor. The first goal is 4.

Student 4 moves 2 cubes before the others even are ready to play.
Student 4: "*2+2*" and tries to select the 2 cubes (but there is no corresponding card).
Student 4: "*It doesn't work, I can take 3…* " (and moves one more cube into the area).
Student 4: *Now it works*" (the card 3 is selected).
Student 3 moves 1 cube into the building area.
Student 1 selects and they receive one star.

Proc. 9th Intl Conf. Disability, Virtual Reality & Associated Technologies
Laval, France, 10–12 Sept. 2012

The next goal is also 4.

Student 4: "*take 3 again*".
Student 1: "*no, no, no, don't*"
Student 4: "*why?*"
Student 1: "*Doesn't work he has no 3, has no 2, must take 1*" (explains why not any combination will do)

The next turn, student 4 seems to have understood and regains the initiative. The goal is 8.

Student 4: "*I know! I can!*" (and puts the first 3 cubes in the area)
Student 2: "*Hey, I never get to help*"
Student 4: "*Hallo, here take!*" (and gives the 4th cube to his play mate to build on top of the others)
Student 1: "*Take 4!*" (he is directing his team mate)
Student 3 builds 4 cubes on their side
Student 4: "*Now [name of student 1]*" meaning that he should select the 4.
Student 1 selects the 4 and they receive another star.

The game was finished and most matching pairs found.

5.5 Progression data from strategic games

Students 1-5 also played some strategic games, which are more advanced than the Match-Pair games since in these game several choices can yield equal score *this* turn, but be more less strategically good either to block the opponent (if competing) or help the partner (if collaborating) *next* turn. Hence, it is not enough to consider one-step ahead, but several in order to play very well. Progression data for participants 1-5 was extracted from the game log. In figure 4, we can see calculated trend lines of "goodness values" from two different strategic games. A goodness value denotes how good a card choice *is compared to the alternatives* including strategic value, i.e. it measures how good the player performs in a given situation, irrespective of how good the cards are or the other player's performance. Here, we are mainly interested in trend lines, which is a measure of a player's *in-game progression* i.e., whether a player improves their strategic game play over time or not.

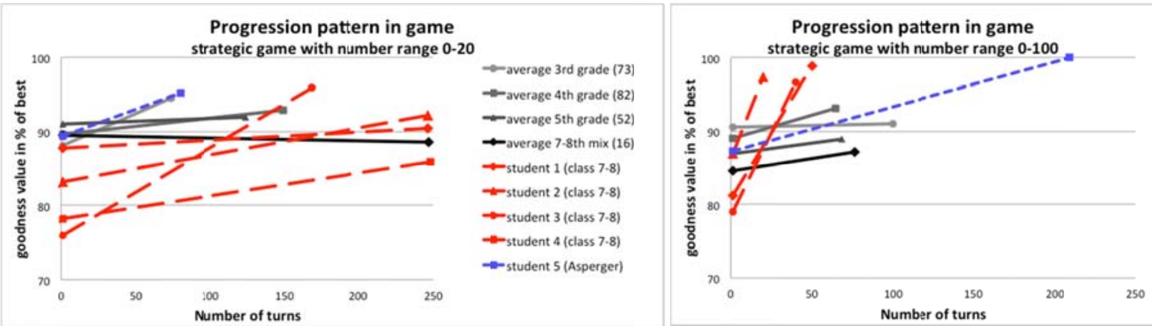

Figure 4. *Progression data from strategic games comparing mainstream and special education students.* The unbroken grey trend lines denote the average trends for 73 3rd graders, 82 4th graders, 52 5th graders and 16 7-8th graders respectively in the two graphs. The red dashed lines are study participants 1-4, and the blue dotted line is the student 5. The y-axis denotes number of turns totally played in the game, so in the left graph the 7-8th graders and student 1, 2 and 4 played about 250 turns, whereas the 3rd graders and student 5 only played about 75 turns in average. Student 3, who started lowest at 76 and ended at 96, made the greatest improvement. The 3rd graders and student 5 also made good progressions ending up at 95 in fewer turns since they started at a higher level. Student 1 improves in the same rate as the 4th graders, but start out a bit lower. Student 2 and 4, who are playing partners, improved at the same quite good rate but at different levels. As a comparison, a goodness value of 97-98 means excellent game play (which is hard for any human) whereas random choice (i.e., only guessing) will yield a goodness value of almost 70. Hence student 3 and 5 reached a very good level of their strategic game play at the simplest games with number range 0-20.

The rightmost trend graph shows a similar but more difficult game in number range 1-100. Here, only student 5 played much, he had an improvement rate similar to the simpler game, and theoretically finished at perfect play (goodness 100). The other study participants only played 20-30 turns each, which means that the trend lines are not very reliable but they indicate greater improvements than the mainstream trends here too.

5.6 Teachers' assessments of student 1-5 progressions (from teacher interview)

The first 8 months of game play students1- 4 mainly played the strategic games up to 10 (the Match-Pair games were not yet designed then), often competitively. There were not much communication and discussions. Student 1 and 3 got very engaged when they learned about the teachable agent, and that was when they started to talk during game play and they even wanted to play at home. When the new collaborative Match-Pair games arrived, the focus turned to these games for communication purposes. In the beginning the students simply were not able to collaborate; they refused to look at each other or at the board, and selected cards blindly. The teachers forced them to talk and to look, and the more they succeeded, the better they played. "*Student 4 is the one who gained most in communication skills, the improvement is just amazing*". Today, he can stand in front of the white board and reason about the solution: "*You can take 7, and I will take 2*".

"The game is the reason [student 4] is able to do math, and has gained self-confidence. [Student 2] now knows the numbers up to 10, and has also gained self-confidence, which in turn gives some self-respect. Hence they talk since they are worth listening to. The game was not popular in the beginning. Today when we mention the game, they smile. Every Thursday they play. They really have a hard time learning, but the game has been fantastic for each of these 4 students."

Student 3 gave evidence of a reflective and advanced thought a week after the science centre visit. He asked the teacher "*How could the screen know how many cubes I had placed there?"*. The teacher replied that it had to be the proper number of cubes, and the response made him angry because of course he knew that but he wanted to know how it worked and how this was possible at all [technically]. The teacher didn't know the answer to that question, but was very pleased of the student's advanced thought.

"I [the SE teacher] think that these students' low abilities is about having failed so many times so there is no self-confidence left at all. Grouping such students together is not good; there is normally no growth in such a group. Playing this game is an exception when growth actually occurs."

The challenge for student 5 was to act as a tutor, i.e., to scaffold the other students to some mathematical understanding. He managed this beyond the teachers' expectations, for instance asking "*Are you sure or should you look at the cards once again?*" instead of telling them they are wrong. Afterwards he spontaneously praised them: "*You have done really well today*", and when the other students had left he returned to the teacher and said: "*This was an experience, I thought they were all really stupid all the time, but they do understand something!"* It was the first time the teacher heard him able to, in retrospect, articulate such reflection over other people's behaviour as well as his own judgement of them. This student has developed from a non-communicating individual to a mainstream graduate in only 3 years.

5.7 Benefits advocated to game by the teachers (from teacher interviews)

The teachers brings forward the following properties of the game as particularly beneficial for student with learning disabilities (as cited from the interview): *1) That the mathematics is represented without digits and symbols, because after many year of failures digits and letter are closely connected to negative thoughts and expectations of more failures. 2) That everything is visual and they [the students] see what it is. 3) That they can see the carrying, see the blocks move one by one and see the actual packing when it transforms into a 10-box instead – what an aha-experience! 4) When they see the addition operate, e.g., 4+3 actually becomes 7 blocks, they understand. 5) That negative integers are included from start and that adding two negative numbers is the same as adding two positive. 6) That it is a computer game, with a motivating reward system (the stars). 7) The teachable agent, some students really liked to train the agent and be on the top lists. 8) That it is something else than a math book, and that it is something different to all other quite similar math pedagogy. 9) That it focus on understanding, not computation procedures. As soon as you do not understand, math becomes difficult and boring. For students understanding math, it is their best subject, and for students not understanding math, it is the worst.*

6. DISCUSSIONS AND CONCLUSIONS

The use of the game differs between special- and mainstream education. In special education, the game is used in groups of 3 or 4 students together with one special teacher and sometimes an assistant. Groups are equipped with, and frequently use an interactive white board. The whiteboard is used for all game play, and is essential according to the teachers. The teacher(s) are constantly present and ready to scaffold and guide the students during game play if needed. The situation in which student 5 conducted his game play was different; it was a mixed class of 16 students with 8 from special education and 8 from mainstream class and they used ordinary computers. In mainstream classes, there are normally 20-30 students, one teacher, and depending on the availability of computers they play all at once or in half class, in pairs. If there is a whiteboard, it is mainly used for introducing new games or particular features whereas students play on the computers.

The learning situation differ from mainstream education in the respect that special education teachers seems more influential and more integral to the students' learning processes and therefore more important. Scaffolding and guidance that can be beneficial to mainstream students may be necessary for special education learners in order to progress, as indicated by the frequent and prolonged introductions of new games and features. Studies reveal how effective teaching becomes when engaged teachers use content-rich instruction that is carefully crafted and responsive to students' diverse needs (Brownell et al., 2010). Hence, a combination of dedicated teachers using challenging and motivating instruction methods can empower students with disabilities to reach higher levels of learning.

Regarding mathematical achievements, the results show that student's number-sense can be improved by game play (Student 2 and 4 both point-counted even small quantities at start but showed fluency in identifying numbers up to 10 in the end, for instance when moving the cubes at the science centre). The reason for such improvement may include the frequent practice of number sense during game play (at least 4 cards should be evaluated in each turn) and the approximate nature of discriminating among alternatives, which is known to contribute to conceptual understanding and number sense (Rousselle & Noël, 2008).

The comparison of mainstream versus special student performance data from the Match-Pair games reveals that *in-game performances* of special students were higher (above 5th grade level) than *traditional-math performance* (below 3rd grade level). One reason can be the graphical representation of numbers. According to Kilpatrick et al. (2001) a good representation can assist students in undertaking mental, mathematical computations previously unavailable to them. Another reason can be the recommendation of Evans (2007) to provide strong and explicit links between procedural proficiency (anticipate the effect of a cards) and conceptual understanding (graphical representation) which is present in every choice in the game.

When considering progression in the strategic games, the comparison revealed that the special education students as expected started at lower levels than mainstream students but that they progressed more, so that 4 of 5 students ending levels were comparable to the mainstream students' averages. However, the progression required more game play. Skilled learners quickly develop efficient strategies whereas students with learning difficulties in mathematics often acquire these strategies at a slower rate (Evans, 2007).

In contrast to mainstream students who usually are enthusiastic about new computer games, the special education students were all very reluctant at start. It took time, but once they started to engage, their motivation and engagement seems to be higher than the engagement of mainstream students. Their endurance in watching others play or play the same game over and over again, and their enthusiasm about every star, was apparent during observations. They seem to try harder, and the game logs support this perception. One motivational aspect can be the non-failing learning situation. Another aspect can be that the game provides a stimulating activity (i.e. strategic play and reasoning) at their mathematical level (perhaps only up to 10). Finally, we have seen that acting an expert (i.e. teach the agent) is highly stimulating for many students.

The gain in mathematical self-efficacy demonstrated by student 4 in the science centre compared to the beginning, was radical. The progression from speaking only three words and having to point-count three objects to directing and reasoning with the others during the collaborative game play is an amazing improvement for one year. Student 3 who improved his strategic game play from the lowest level to a very good level made a more silent progression, yet impressive. His engagement and self-efficacy became evident at the science centre, where he built almost everything on one side without hesitation. His question about the augmented reality system is evidence of advanced thinking beyond the teachers' expectations.

Besides the mathematical learning, we have seen gains in communication and collaboration skills, in particular in peer-to-peer collaboration, as evident from the science centre observation. Students 6-8 also improved their collaborative skills during sessions. Acting an empathic tutor, is an achievement for a person with Asperger syndrome, and he judged this day to be the best of all in 9th grade. Other proofs of enjoyment using the game include singing and laughing while playing, wanting to play at home and in school, and engaging more and longer than with other activities.

To conclude, the study showed that our arithmetic game can be used effectively by dedicated teachers in special educations: 3 students showed great learning gains in mathematical understanding, strategic thinking or communication skills, 2 students showed good mathematical learning gains at their respective levels, and 2 students are progressing but have not played long enough yet. The game has failed, so far, to engage one student. The augmented reality version of the game seems promising to further engage and foster deep number sense in students with intellectual disabilities.

Acknowledgements: This research is supported by Wallenberg Global Learning Network, the Swedish National School Board and the Swedish National Agency for Special Needs Education and Schools**.**

7. REFERENCES

Berch, D.B., (2005) Making Sense of Number Sense: Implications for Children With Mathematical Disabilities. In Journal of Learning Disabilities, Volume 38,: 4, 2005, pp 333–339.

Butler, F.M., Miller, S.P., Lee, K and Pierce, T. (2001). Teaching Mathematics to Students With Mild-to-Moderate Mental Retardation: A Review of the Literature. In Mental Retardation Vol 39(1), pp 20–31

Butterworth B. (2005). The development of arithmetical abilities. In The Journal of Child Psychology and Psychiatry. 2005 Jan; 46(1):3-18.

Brownell, M., Sindelar, P., Kiely, M., and Danielson, L. (2010). Special education teacher quality and preparation: exposing foundations, constructing a new model. In Exceptional Children Volume 76(3).

Carpenter, T. P., Fennema, E., & Romberg, T. A. (Eds.). (1993). Rational numbers: An integration of research. Hillsdale, NJ: Lawrence Erlbaum Associates.

Case, R., & Okamoto, Y. (Eds) (1996). The role of central conceptual structures in the development of children's thought. Chicago: University of Chicago Press.

Dowker, A. (2005). Individual differences in arithmetic: Implications for psychology, neuroscience and education. Hove, UK: Psychology Press.

Evans, D. (2007). Developing Mathematical Proficiency in the Australian Context : Implications for Students With Learning Difficulties. In Journal of Learning Disabilities, Vol 40:5, 2007, pp 420–426.

Grégoire, J and Desoete, A (2009). Mathematical Disabilities—An Underestimated Topic? In Journal of Psychoeducational Assessment Vol 27:3, 2009, pp171-174

Garnett, K. (1998). Math Learning Disabilities. Division for Learning Disabilities Journal of Council for Exceptional Children. Available on http://www.ldonline.org/article/Math_Learning_Disabilities

Geary, D. C. (1999). Mathematical disabilities: What We Know and Don't Know. In Learning Disabilities Online, Available at http://www.ldonline.org/.

Geary,D.C.(1993).Mathematical disabilities: Cognitive, neuropsychological, and genetic components. Psychological Bulletin, 114, 345–362.

Gersten, R., & Chard, D. (1999). Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities. The Journal of Special Education (1999) 44:18-28.

Gindis, B (1995). The Social/Cultural Implication if Disability: Vygotsky's Paradigm for Special Education. In Educational Psychologists, 30(2), pp 77-81.

Grant, G, Goward, P., Ramcharan, P. and Richardson, M. (2010). Learning Disability: A Life Cycle Approach to Valuing People. McGraw-Hill International. p. 43-44.

Jonassen, D., & Ionas, I. (2008): Designing effective supports for causal reasoning. Educational Technology Research and Development, 56(3), 287--308

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.

Lock, R.H. (1997). Adapting Mathematics Instruction in the General Education Classroom for Students with Mathematics Disabilities. In LD Forum: Council for Learning Disabilities www.ldonline.org

McPartland J, Klin A (2006). "Asperger's syndrome". Adolesc Med Clin 17 (3): 771–88.

Miller, S.P. & Mercer, C.D. (1997). Educational Aspects of Mathematics Disabilities. *Journal of Learning Disabilities,* Vol 30 (1). 47-56.

Moreno, R., & Mayer, R. E. (2005). Role of guidance, reflection and interactivity in an agent-based multimedia game. *Journal of Educational Psychology*, *97*(1), 117-128.

Munro J., (2003). Dyscalculia: A unifying concept in understanding mathematics learning disabilities. In *Australian Journal of Learning Disabilities*, 8, (4), p 25-32

Nilsson, A. & Pareto, L. (2010). The Complexity of Integrating Technology Enhanced Learning in Special Math Education – a Case Study. Sustaining TEL: From Innovation to Learning and Practice, Lecture Notes in Computer Science, 2010, Volume 6383/2010, pp 638-643

Pareto, L., Arvemo, T., Dahl, Y., Haake, M. and Gulz, (2011). A. A Teachable-Agent Arithmetic Game's Effects on Mathematics Understanding, Attitude and Self-efficacy. In Lecture Notes in Computer Science, 2011, Volume 6738, Artificial Intelligence in Education, Pages 247-255

Pareto, L. (2005). Graphical Arithmetic for Learners with Dyscalculia. In Proceedings of the 7th international ACM SIGACCESS conference on Computers and accessibility (Assets '05). ACM, New York, NY, USA, 214-215.

Ranpura, A. (2000). Number Blindness: A Hidden Challenge for Mathematics. Scientific Learning. Available online: http://www.brainconnection.com/.

Rousselle, L & Noël, M-P. (2008). Mental Arithmetic in Children With Mathematics Learning Disabilities. In Journal of Learning Disabilities, Volume 41:6 2008 pp 498-513

Shneiderman, B. (2000). "Universal usability", Communications ACM, vol. 43, no 5, 84-91. ACM Press.

Vygotsky, L.S. (1978). Mind in society. Cambridge, MA. Harvard University Press.